VISION-AND-MOTION-BASED SPATIAL CHARACTERIZATION OF OPEN SPACE:

a study on its structure, boundary and meaning in architecture

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ABSTRACT
Spatial analysis from human perspective is particularly difficult in describing people’s perception objectively. Quantitative analysis seems to be a good measurement of human feelings. However, due to the ambiguous transforming process and the complex multivariable analysis, the correlation between the data and human experience are often weakened. In this context, we first turn to the very nature of human vision. Starting with how eyes work and what is important to human field of view, we find a new perspective of understanding open space which is the three-dimensional space between buildings in brief. By comparing the characteristics of an observer’s location and his vision in a dynamic way, we introduce solid angle as a measurement of not only human field of view, but also human visual impression as well, which connects closely the subjective visual perception with the scene we see in the objective world. Then, by proposing a concept of “equi-visual set” where one object seems to be the same size as another, we put forward not only a simplified mathematical model with spherical objects (or “buildings”), which is available easily by scripting, but also a new method of calculating the neighbourhood which represents the space belonging to a certain sphere. This model reveals a basic structure of open space which consists of enclosed territories and shared boundaries within an artificial bounding box containing all the spheres. Moreover, it helps us, by mapping and animation, in explaining how this neighbourhood relates to the human visual perception and the publicity of open space in a broader sense. Finally, in consideration of the two-dimensional track of human, ignoring the terrain, we generalize a new model with 2.5 dimensional convex hulls in place of the one with spheres, which represents the buildings in a more real way. By sampling in the eye-level plane, we achieve a spatial section of open space with two-dimensional neighbourhoods, providing a new methodology for spatial analysis while revealing its significance in architecture.

KEYWORDS
Vision and Motion, Solid Angle, Equi-visual Set, Neighbourhood, Boundary

1. INTRODUCTION:
1.1 Background
Since Benedikt introduced the concept of isovist into architecture space analysis in 1979, various researchers developed this perceptual measurement framework on learning and analysing the relationship between space and human visual perception, trying to link the physical world with human feelings. Some of them focus on the computer simulation of 2D and 3D isovist (Batty, 2001; Truner et al, 2001; Fisher-Gewirtzman and Wagner, 2003; Yang et al, 2007), while others pay more attention to analyzing the photographs or the moving images taken from the axis of urban street (Porta and Renne, 2005; Cooper and Oskrochi, 2008; Ding, 2011.).

However, few of these studies do focus on what people actually see and how they really sense the space through their eyes (not through a video camera). First, taking isovist, the set of all points visible

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from a given vantage point in space (Benedikt, 1979) for example, even though it shows us ‘How far can we see, and how much can we see?’ in space (Batty, 2001), it does not tell us ‘How much can we feel from what we see and which entity of an environment around us inspires our certain spatial experience?’. The relevance of vision, spatial feeling and environment is still not clear. Second, photographs and moving images are mainly based on the rectilinear projection. This way of projection (also known as perspective) brings distortion on the edge of image, especially when the lens of camera gets shorter focal length. That’s why people often feel cheated by the beautiful pictures on magazines when they reach the place where the photographs were taken. For this reason, researchers prefer to select a standard lens (like 50mm standard lens) to simulate the central part of human field of view. This substitution leads to a lack of peripheral vision of human eye and finally results in an inaccurate conclusion (Porta and Renne, 2005; Cooper and Oskrochi, 2008). Third, for computational convenience, the movement and the location of observer are restricted to the central axis of a street or to a grid with a fixed distance (Yang et al, 2007). This simplification goes against with our everyday behaviour. No one would walk along the axis of a street to feel the space, while no one would experience the public space by ‘jumping’ from one node of the grids to another.

Another problem of the above spatial studies is the process of transforming visual information into data for analysis. The quantitative process adopted seems to be a good measurement for human perception, but sometimes it makes the perception too complex to understand. The statistical analysis and the multivariable analysis do not always tell the inherent rules but ambiguity. Sometimes the abstract analysis confuses us rather than the original photographs or images, which makes it hard to see the correlation between data, charts and visual feelings directly. In this context, we turn back to the very nature of human vision and motion, trying to develop a new method, wholly based on how people feel about the space, to understand the effect of open space between buildings. In this paper we:

- First review the characteristics of human vision and motion, learning how people see things and how people change their locations while walking.
- We then discuss the structure of 3D space with a mathematical model of spheres, which is a theoretical model based on human vision. By proposing the concept of “equi-visual set” where one sphere looks the same big as another, we find a new way to calculate the neighbourhood of spheres in 3D space, which shows the basic visual spatial structure of 3D open space.
- We then develop the theoretical model into a model of 2.5D convex hulls, which represents the buildings in a way which is closer to reality. By calculating the section of the neighbourhood of convex hulls on the visual plane, we realize a 2D spatial mapping of 3D open space under the consideration of human vision and motion.
- We then apply this method to several different virtual open space samples in the same size, to show how artificial buildings affect our visual perception of open space.
- We finally conclude the two experiments’ results and propose their potential application in space design.

1.2 Characteristics of human vision

How do people on earth feel the environment through their eyes? First of all, figure 1 shows the maximum field of view of human eyes. It has a horizontal range of 188° and a vertical range of 120° to 135°. Considering the up-and-down movement of the head, a person standing still has a view more than 2π sr in solid angle, which means more than half of the environment can be seen and felt by people without turning around. This is why we can perceive “front” and “back” in 3D space.

Secondly, the entity itself, its image taken by camera, its projection on curved retina (retinal image) and its image in human brain from optic nerve are all different. Jon Minjon indicated this proposition and suggested photographs taken by camera with fisheye lens as a substitution of human field of view (Jon Minjon, 2004). In spite of the distorted appearance, with a hemisphere view (2π sr in solid angle), fisheye lens makes it possible to record the extent of view through human eyes (but not what people really see). According to this conclusion, even though it seems impossible to describe the same image of what people really see through their eyes in a two-dimensional way, we can still be sure that the hemisphere view collected by our eyes could not and should not be diminished in spatial cognition.

Thirdly, the proportion of an entity in human field of view determines its visual size. It is sometimes understood as the phenomenon that an entity appears smaller as its distance from the observer.
increases. Actually, the visual size of an entity is attributed to the solid angle rather than its distance from the observer. The top illustration of figure 2 and equation (1) shows the relationship between $L$ and $S$. $L$ represents the distance from the centre of a sphere (point $O_1$) to the observer’s eyes (point $O$) and $S$ represents its solid angle in human field of view. The radius of sphere $O_1$, the radius of visual sphere and the angle of view in a section plane through $OO_1$ are written as $r$, $R$ and $\theta$.

\[
(1) \quad S = \frac{2\pi R h}{R^2} = 2\pi \left(1 - \frac{r^2}{L^2}\right)
\]

\[
(2) \quad S = 2\pi \left(1 - \cos \frac{\theta}{2}\right)
\]

From the equation (1), on one hand, the bigger the length of $L$, the smaller the solid angle that the sphere occupies in the field of view and the smaller the sphere’s visual size in human eye; on the other hand, the bigger the radius of the sphere $r$, the larger the solid angle of sphere in the field of view and the bigger the sphere looks like. As a result, something looking big does not mean it is really big. On the contrary, something seeming far does not say it is indeed small. These phenomena give us the ability to comprehend the spatial concept of near and far as well as big and small. The equation (2) also shows the positive relevance between $S$ and $\theta$, which helps us evaluate the solid angle of spherical entity in a section plane.

![Field of view of the human eye](image1)

**Figure 1**: Important human visual data

1.3 Characteristics of human motion
The most important issue of human motion is that people could almost only move on a 2D surface, due to the gravity of the earth (Zhang and Cui, 2002). Here, a 2D surface does not need to be a flat plane. It represents a continuous surface in $R^2$ space (David Rutten, 2007). It can be a sphere, a mountain field and also a Euclidean plane. The moving of human body on a 2D surface results in a similar moving pattern of human eyes. It is also a 2D surface which is about 1.6m higher than the altitude of ground according to the straight height from eyes to standing position (Tilley, 2008). In the real world, most settlements and cities built up on plains rather than mountain lands for further development convenience. In addition, a flat ground helps us simplify the calculation of the mathematical models. Taking these into consideration, we choose a flat plane as the ground to start our experiments.

The second key point of human motion is the step-by-step walking pattern. People always alternate their two feet back and forth while walking, and their vision of the environment surrounding them also changes following the positions of their steps. So, it is reasonable to choose footprints on the walking track to simulate the moving visual perception of humans. The average step length, which is about 0.64m (Architectural design dataset, 2017), is thus an important parameter for the simulation.

2. DATASETS AND METHODS

Based on the above characteristics of vision and motion, we developed two spatial models: one is the model of spherical entities and the other is the model of 2.5D convex hulls. The first one helps us to understand how entities act on spatial structures in a broader sense, while the second one stimulates a more real virtual built environment by two-dimensional mapping of spatial structure.

2.1 Spatial experiment with model of spheres

The reason why we chose sphere is mainly based on two considerations. First, it’s easy to describe the size and location of a sphere with only two parameters: its radius and the coordinates of its center; Second, the projection of a sphere from a given point is always a circle, so it is convenient to calculate its solid angle and to compare the visual size of two spheres with different radius. Before we start to discuss the mechanism of the open space’s structure, let’s imagine three virtual scenes in our mind first.

Scene 1: Imagine you are in an empty 3D space, like somewhere in the universe. In this space, you can make free space walk in all direction without gravity and observe the world in any possible way. However, even with all your efforts, what you can see and feel is only a void. Nothing seems different.

Scene 2: Suddenly, a sphere ($S_1$) appears in front of you. It changes the world thoroughly. When you try what you’ve done in Scene 1 again, you’ll find the sphere’s size begins to change: when you go towards the sphere, it becomes bigger; and when you walk away, it becomes smaller. When you get close enough to the sphere (assuming the sphere has an absolute bigger size than your body at least), you’ll feel enclosed and the degree increases while you get closer to it. You can also feel that $S_1$ is disappearing, when you go far far away from it. Its influence on you gradually fades out.

Scene 3: Now, what would happen when another sphere ($S_2$) with a different size from $S_1$ appears beside the original one? All that you see can be summarized in three cases. The first one is two separate spheres; the second one is two ‘intersecting’ spheres and the third one is a single sphere. We can also describe what you can see differently in Scene 3. There’re still three cases: $S_1$ looks bigger than $S_2$; $S_2$ looks bigger than $S_1$ or $S_1$ looks as big as $S_2$. For the first two cases, You’ll get a feeling similar to the one that you experienced in Scene 2: When one of the spheres becomes bigger and gets closer to you, its impact on you increases with its growing solid angle in your field of view; In contrast, the smaller and further sphere has a weaker influence on your spatial impression with a smaller solid angle in your sight. For the last case, when two spheres appear to the same size in your view, there’s even a moment that you can’t tell the difference of them without marking or coloring $S_1$ and $S_2$.

Now let’s get back from the thought experiment to see what really happen when an observer experiences these three scenes. In Scene 1, nothing changes in the field of view when the observer moves and looks around until the first entity $S_1$ comes into sight in Scene 2. It shows us that the entity in space gives birth to the observer’s spatial experience with the concept of near and far as well as big and small. In Scene 3, the observer starts to get a new impression of space between entities for the first time. As the viewpoint moves around, $S_1$ and $S_2$ keeps changing their positions and visual size
alternatively. When the two spheres get the same visual size, they seem to have the same power on the observer’s spatial impression. These prompt us to ask a question: Is there a set of points where the observer always gets the same visual size impression of the two spheres?

The answer is yes. The bottom diagram on figure 2 shows the mechanism of this set (named $E$) in a section plane through $O_1O_2$. $O_1$ and $O_2$ represent the center point of $S_1$ and $S_2$, while $r$ and $R$ represent the radius of $S_1$ and $S_2$. $o$ is both the origin of Cartesian coordinate system and the midpoint of $O_1O_2$. $L$ is the distance between $O_1$ and $O_2$. Point $S(x, y)$ is a typical point of set $E$. ‘The same visual size impression’ can be interpreted as $S_1$ and $S_2$ having the same solid angle from viewpoint $S$. It also means $\theta$, the angle of view of $S_1$ and $S_2$ from point $S$, is equal too. So we have the equation (3):

$$ \left( \frac{r}{R} \right)^2 = \frac{(x + \frac{L}{2})^2 + y^2}{(x - \frac{L}{2})^2 + y^2} $$

Let $p = R / r$, then we get a section curve of set $E$ as follows
The equation (4) is a circle function with radius \( R_v \) of whose center’s \( O_v \) coordinates is \( (-\frac{pL}{p^2 - 1}, 0) \). It indicates that set \( E \) is a spherical surface bounding the smaller sphere \( S_1 \). When the two spheres have the same radius, the set \( E \) becomes a bisector between them through the midpoint of \( O_1O_2 \), which can also be treated as a sphere of infinite radius.

The spherical surface of set \( E \) (named surface \( B \)) separates the 3D space into two parts: the inside of surface \( B \) (named set \( N_1 \)) and the outside of it (named \( N_2 \)). It’s easy to prove that \( S_1 \) always looks bigger than \( S_2 \) in observer’s sight when viewpoint \( S \in N_1 \), vice versa, \( S_2 \) looks bigger than \( S_1 \) when \( S \in N_2 \). This makes it possible to give a new way to calculate the neighbourhoods of 3D open space based on human visual perception in the model with spheres. \( B \) is actually the boundary of the neighbourhoods between \( N_1 \) and \( N_2 \). In these two subspaces, the observer gets typically different visual impression respectively: a certain sphere always gets bigger visual size and spatial influence on the observer while he or she is in its corresponding subspace.

By scripting in Rhinoceros, we try to prove this reasoning with a virtual experiment, which involves two separated spheres of different size and locations. Figure 3 shows the process in four illustrations. Each of them is a screenshot with two or four viewports named after their corresponding view. In the first illustration, the blue spheres marked \( O_1 \) and \( O_2 \) are separated by the magenta sphere (set \( E \)). When we set a camera on any given point (named \( C \)) on the magenta sphere, and set its target point on the angle bisector of \( \angle O_1CO_2 \), we get a pair of ‘same looked’ blue spheres in the second illustration. Then we pick up one latitude line (the cyan circle) from the magenta sphere, to generate a series of viewpoints of camera in equidistance on the cyan curve and the corresponding camera target points on the green curve (the set of bisectors of \( \angle O_1CO_2 \)) in the third illustration. The last illustration shows the current camera’s position in ‘Ground_Plan’ viewport on the left and the view with two ‘same big’ spheres through camera \( C \) on the right. After setting cameras on the cyan latitude line, we make an animation of 100 frames to stimulate the experience of spacewalk on the latitude. Figure 4 shows ten key frames with its numbers in order below. Although the camera view has strong distortion on the edge of the viewport, the pictures we see from key frames are two symmetric blue circles (the projections of two spheres) of equal visual size all the time. It proves the previous theoretical derivation from equation (4).
Referring to the mathematical concept of ‘equidistant set’, we define set $E$ as an ‘equi-visual set’, which is a set that each element has the same solid angle from two or more entities. This definition does not only describe a boundary set between two or more given entities, but also says that the observer would get the same spatial image from the adjacent entities when he looked around from the equi-visual set. It is proposed to be a new method of spatial subdivision based on human spatial perception with their eyes.

What would happen when there’re more than two spheres in this model? Let’s suppose a 3D space with $n$ isolated spheres in random size. For any given sphere, we can compute its neighborhood, or the subspace that belongs to it, by finding the intersections of its neighbourhoods with the other $n-1$ spheres iteratively. Figure 5 shows two of this kind of spatial neighbourhoods with 50 isolated spheres in two bounding boxes which contains them all: the spheres in the right one are plotted in a cubic space randomly, while the spheres in the left column are arranged on a horizontal plane randomly. We call the green bubble-shaped space unit ‘cell’. Each cell belongs to only one sphere and each sphere is only wrapped by its corresponding cell. From the ‘section’ and ‘internal structure’ in figure 5, we can clearly see the structure of 3D open space among the spheres. In any given cell, the sphere in it always appears to be the ‘biggest’ one when compared with the others, and the space inside of the cell seems to be ‘controlled’ by the sphere. If the observer’s eyes travel on the boundaries of the cells, he or she will find that all the spheres in the adjacent cells have the same visual size when he or she looks around. We can also understand this phenomenon as a spacewalk in ‘public’ space. It implies that the distance between a specific viewpoint and the boundaries of cells can be treated as a measurement of privacy. The closer the observer gets to a certain sphere from its cell’s boundary, the more private the observer feels.
Figure 5: Spatial structure of spheres in 2D plane and 3D space
2.2 Spatial experiment with model of 2.5D convex hulls

Even though the model of spheres reveals the basic spatial structure of open space based on human visual perception, it is still quite different from the situation in the real world. For this reason, we introduced a developed mathematical model with 2.5D convex hulls, representing the buildings of settlements and cities in a more real way. A 2.5D convex hull is a solid which is extruded from a two-dimensional convex polygon and can be described with a 2D convex polygon and its height.

Figure 6: Diagrams for calculating the solid angle of the convex building from the given point E

Figure 6 shows how to compute the solid angle of a 2.5D convex hull from a given point $E$ (on which the observer’s eyes are when he stands on point $X$). The length of $EX$ is equal to the eye height which is 1.6m high (as we’ve discussed in the first part of this paper). The green pyramid space represents the isovist (or visible space) of a given 2.5D convex hull (or building) from point $E$. We add a cyan visual sphere of any given radius $R$ to intersect with the volume of the isovist. The intersection on the visual sphere is a spherical surface in cyan as the forth illustration shows. If we observe the 2.5D convex building from point $E$, we’ll find that the cyan spherical surface, which is also a spherical polygon, has the same contour with the visible vertical faces bounded by a closed polyline in blue and
Let’s suppose surface $B$ as the equi-visual set (named set $E$) or the 3D neighbourhoods’ boundary of $B_1$ and $B_2$ (which represent two isolated 2.5D convex buildings). It won’t intersect with $B_1$ or $B_2$ in any possible location of set $E$, simply because if $B$ has any intersection with $B_1$ or $B_2$, there must be at least one point on $B$ which is inside of one certain entity and outside of another (for $B_1$ and $B_2$ are isolated), and the solid angle of the entity containing this point will have a bigger value of $4\pi$ sr than the other one. Similar to the situation in the model with spheres, surface $B$ could only be either a closed surface or a non-closed surface between $B_1$ and $B_2$. Whatever it is, if we make a section in the visual plane, we’ll get a curve, either open or closed, from surface $B$. When the observer’s eyes move on this curve, he’ll get an impression that $B_1$ is as ‘big’ as $B_2$, which means the observer gets same the solid angle of $B_1$ and $B_2$ from his viewpoint. So, it is reasonable to calculate this 2D section of surface $B$ instead of itself.

According to the above analysis, we propose a method to generate the projection of the equi-visual set on visual plane by introducing the greedy algorithm. The whole process is split into two steps:

First, the top illustration in figure 7 shows the process of finding the first point on the section curve of surface $B$ (the dashed line in red, named $C_t$). $N_1$ and $N_2$ are the 3-D point locations on $B_1$ and $B_2$ where they are closest to each other. There must be a point $F_0$, which is located on both segment $N_1N_2$ and curve $C_t$ for the same reason we’ve discussed in the last two paragraphs. Although our intuition says that the closer we get to a certain entity, the bigger the solid angle that it occupies our field of view, it’s still difficult to fully prove this phenomenon due to the complicated situations which are changed with the observer’s locations.

Fortunately, we may not need to do this before we start searching $F_0$. Following the greedy algorithm, we can gradually access to the target point by making the locally optimal choice in each step. For example, we choose $T_1$, the midpoint of segment $N_1N_2$, as the temporary candidate of $F_0$. Then we compare the solid angle of $B_1F_0$ (named $S_{a1}$) and the solid angle of $B_2F_0$ (named $S_{a2}$) from point $T_1$ by using equation (5). If the difference between $S_{a1}$ and $S_{a2}$ are bigger than the tolerance of solid angle (named $T_s$), then we try to move the viewpoint to the midpoints of two segments ($N_1T_1$ and $T_1N_2$), and make the comparison of their solid angles again. If one of the newly calculated difference between $S_{a1}$ and $S_{a2}$ is the minimum and still bigger than $T_s$, we then pick it as a new candidate and continue the process again until the difference is smaller than $T_s$.

Another exit signal of this process is the tolerance $T_d$ (which represents the minimum afforded distance between two adjacent midpoints $T_1$ and $T_2$), which can effectively diminish the computing operations when $B_1$ and $B_2$ are close enough to each other relative to their sizes. It is because the solid angle of these two entities from the viewpoint are highly sensitive with $L$, which is the distance between the viewpoint and any given entity of the two, according to equation (1). In figure 7, the candidate of $F_0$ changed from $T_1$, $T_2$ to $T_3$ in sequence, until $| S_{a1} - S_{a2} |$ is smaller than $T_s$ or $d$ (the length of segments $T_3T_2$) is smaller than $T_d$. 

It is really hard to calculate the equi-visual set of a pair of isolated entities, even when they’re simplified 2.5D convex hulls, because we cannot easily find a mathematical function to describe this set as what we’ve done in the model of spheres. However, further observation indicates that only visible vertical faces of a 2.5D convex building contribute to the solid angle from a given point, when both the observer and the building are on the same horizontal plane. It is because the visual plane of the observer is always between the roof and the ground floor. This fact provides us the possibility to calculate the equi-visual set of 3D open space in a two-dimensional way instead of the three-dimensional one. It does not only simplify the calculation, but is also reasonable because, in most cases, people’s visual perception of the environment only stays on the visual plane as we’ve discussed the characteristics of human motion at the beginning.

$$ (5) S_a = A_n / R^2 = (2a_1 - (n - 2) \pi) R^2 / R^2 = 2a_1 - (n - 2) \pi $$

The length of segments $T_i$ between the viewpoint and any given entity of the two, according to equation (1). In figure 7, the candidate of $F_0$ changed from $T_1$, $T_2$ to $T_3$ in sequence, until $| S_{a1} - S_{a2} |$ is smaller than $T_s$ or $d$ (the length of segments $T_3T_2$) is smaller than $T_d$. 

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Second, we can find all the other sample points $F_i$ by walking along the projection of $CE$ on the ground floor step by step. Similar to the process of calculating $F_0$, we follow the greedy algorithm to find the next typical point $F_i$ on $CE$ by locating it on the semi-circle curve $LR$ with a radius of step length in the forward direction. The bottom diagram in figure 7 shows that by changing the location of viewpoint on $LR$ from point $P_1, P_2, P_3$ to $P_i$, which are the midpoints of its corresponding arc $LR, LP_1, P_1P_2$ and $P_3P_4$, point $P_i$ keeps approaching the target point $F_i$ until the difference between $S_{a1}$ and $S_{a2}$ from $P_i$ is smaller than $T_s$. We also introduce $T_d$ (the tolerance of minimum arc length between $P_{i-1}$ and $P_i$) as another exit signal to diminish unnecessary calculations.

In addition to the two steps mentioned above, we still have to consider some details before we start to find the sample points on curve $CE$. Above all, how could we know that we’ve found all the sample points and what should we do if $CE$ is an open curve with infinite length or a big enough closed curve with a huge perimeter? Figure 8 shows three typical cases as follows. Case No.1 on the left shows a closed curve $CE$ in blue and its red sample points numbered in order. We can simply get a signal of finish by comparing the distance between the current sample point $F_i$ and the starting point $F_0$ (named $D_i$) with the step length. If $D_i$ is smaller than the step length, we finish the calculation and draw a proximate closed polyline of $CE$ with a one-dimensional array of sample points.

However, case No.2 in the middle of figure 8 shows a more difficult situation. A closed curve $CE$, far bigger than the one in the first case, has a huge perimeter which takes longer time to calculate all of the sample points. In fact, in most circumstance, it is unnecessary to calculate all the sample points because some of them that is too far away from the entity (referring to its size) does not contribute to the boundary of its corresponding cell at all. So it is reasonable to calculate a partial closed curve instead of the original $CE$ by introducing a bounding frame (named $F$) which includes all of the
entities. If \( C_e \) is totally beyond the boundary \( F \), it means that \( C_e \) intersects with \( F \) at two isolated points (point \( L \) and point \( R \) in black) at least. It is possible to search point \( F_i \) and point \( S_i \) (typical sample point which is opposite to the searching direction of \( F_i \)) in two reverse direction respectively until they meet the boundary \( F \) for the first time. Then we’ll get a three-segment closed polyline curve, consisting of sub-curve \( P_f \) through red sample points, sub-curve \( P_s \) through yellow sample points and sub-curve \( P_b \) which is overlapped with boundary \( F \) from point \( L \) to point \( R \). Any viewpoint inside this closed curve always gets bigger or at least same solid angle of \( B_1 \) compared with \( B_2 \). The frame \( F \) helps us cut off the useless part of the equi-visual set \( C_e \).

Case No.3 on the right of figure 8 shows a particular case. Curve \( C_e \) is an open curve with infinite length, which makes it impossible to calculate all the sample points on it. Nevertheless, we can still introduce the bounding frame \( F \) as what we’ve done in case No.2 to compute a closed curve as a substitution of \( C_e \). This closed curve is also made up of three parts: the first part of polyline extending through red sample points in one direction, the second part of polyline passing through yellow sample points in an opposite direction, and the third part of polyline linking the last two parts and overlapped with curve \( F \). Once again, the three-segment curve as a substitution of \( C_e \) also guarantees the maximum solid angle of \( B_2 \) from any given point inside this curve.

The second detail we would like to discuss is the tolerance (\( T_s \) and \( T_d \)). The minimum angle of view at which people can distinguish between different entities is about 1’ in degrees (Liu, 2011). According to equation (1) and (2), this is equivalent to observing a ball of 1m in diameter, which is about 1719m away from the observer. The solid angle of the ball takes up about 1/47272412 of the visual sphere whose solid angle is 2\( \pi \). It’s a very tiny proportion and is not suitable for \( T_s \) as a signal to cease the calculation. If the ball is 100m away from the observer, it takes about 1/159999 of the whole visual sphere and about 0.57° in the observer’s angle of view. Considering people’s everyday experience, it is appropriate to choose this small enough solid angle, which is about 7.85\( \times \)10\(^{-5} \) sr, as the value of \( T_s \). As a supplementary tolerance for distance between two adjacent candidates of point \( F_i \), the value of \( T_d \) does not have to be too small. Taking into consideration the size of human eye, we assign 1cm to \( T_d \).

Another detail is the step length we’ve discussed in the first part of this paper. In the model with 2.5D convex hulls (or buildings), we set the step length to 0.64m according to human factors. In fact, the above parameters can be changed flexibly under different circumstance. For example, if we want to get a quick calculation, we can enlarge the step length to get an approximate result.

One more important thing worth noticing is that even \( B_1 \) and \( B_2 \) in case No.2 and No.3 are exactly the same in shape and location from top view, they get totally different two-dimensional equi-visual set due to the different heights of the entities. The taller the building \( B_2 \) is, the further the boundary of equi-visual set away from \( B_2 \). This phenomenon is mainly attributed to perspective. Another interesting phenomenon in case No.3 is that when two entities are symmetric along a certain axis or a vertical face, the curve \( C_e \) and the surface \( B \) are exactly the axis itself and the vertical face respectively. It means that when entities are homogeneous in both shape and position, the space they occupy are also homogeneous. We can further explain it as the influence of homogeneous entities on spatial visual perception balanced on the bisector between them.

So far, we’ve achieved the two-dimensional equi-visual set between a pair of 2.5D convex buildings within a bounding frame. From now on, we’ll discuss the spatial structure of more than two entities. Before we begin, we would like to give a description of the experimental environment first.
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Considering the characteristics of 2.5D convex hull, we first draw or import \( n \) closed convex polygons which represent the contours of buildings in plan on the ground floor. The ground floor is bounded by a rectangular frame (named \( F \)) which contains all the buildings and represents the extent of the calculation. Then we add an annotation in each polygon to record the number of floors (named \( N_f \)) of each building. By setting the floor height (named \( H_f \)) with a specific value, we can get a total height of a building from the product of \( N_f \) and \( H_f \). In this model, we assign 3m to \( H_f \) referring to the floor height of real buildings.

Suppose a ground plane with \( n \) isolated 2.5D convex buildings of different size and height on it. Referring to the process of computing the neighbourhoods of spheres in 3D space, we may compute the projection of any given building’s neighborhood (or the subspace belongs to it), by Boolean intersection with \( n-1 \) curves, each of which is a two-dimensional equi-visual set between the given building and one of the other \( n-1 \) buildings on visual plane iteratively.

By scripting in Rhinoceros, we generate six typical virtual samples and its two-dimensional neighbourhoods in figure 9 as below. Each sample has a blue ground plane which is 100m × 100m and contains all the 2.5D convex buildings in orange. Each building in any given sample is located in a corresponding polygon whose boundary are printed in black. Similarly, we also call these polygons ‘cells’. It is worth noting that these two-dimensional cells on the ground are not really the section of the buildings’ three-dimensional neighbourhoods at the ground floor, but the horizontal projection from the section of neighbourhoods on the visual plane. There’re mainly two reasons for this: one is that the section floating at the eye level blurs the relationship between the buildings and their corresponding space cells; the other is because if the observer stands at a cell’s boundary on the ground, his eyes are just at the section of the equi-visual set, where he would get the same solid angle of two or more buildings in adjacent cells.

![Figure 9: the spatial structure of six virtual samples of 2.5D convex buildings](image)

3. RESULTS

In the second part of this paper, we discussed comprehensively two spatial models: the theoretical model of spheres and the more realistic model of 2.5D convex hulls. The first one reveals a basic structure of open space with spherical entities, while the other one bridges the substructure of artificial built environment with our intuitive spatial perception through eyes. Here, we would like to talk more about the calculation results in figure 9. On account of the distortion of perspective and photograph of
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fisheye lens, we only showed the spatial mappings from top view and isometric view. For this reason, we ask the readers to imagine the spatial experience as much as possible in the following discussions.

In sample 1, we see nine same buildings arranged in a matrix, which are very similar to some residential buildings built up in 1980s-1990s in China. According to the analysis of case No.3 in figure 8, we can find similar homogeneous spatial structure in sample 1. In this case, the substructure of the open space is simple and straight. If people walk along the cells’ boundaries, what they see and feel are also homogeneous. This kind of structure brings boring spatial experience.

Sample 2 and 3 shows us two village-like layouts of buildings. The buildings in sample 2 have the same square plan (10m × 10m) but different in heights. The taller building always ‘controls’ more space than the lower one, because the solid angle it occupies from ‘midpoints’ (or the boundaries of two-dimensional Voronoi diagram of the buildings) between them are bigger than the lower one in the observer’s field of view. The buildings in sample 3 are all irregular convex polygons with more than four sides from top view. We can find that the smallest building with seven floors does not ‘control’ the least space due to its height, and the biggest one with seven floors occupies the most space and creates a strong compression on the surrounding buildings due to its biggest volume. If the observer walks in these two virtual environments, he or she will get very different experiences: the former one brings people a feeling of uniform publicity and similar private space due to the similar size of buildings; and the later one gives a hierarchical spatial experience when people walks through sample 3 in different directions because the volume of the buildings varies greatly.

Sample 4 is a very special case in which all the buildings are 24-gons in plan. The pattern of cylinder-like buildings’ neighbourhoods, whose boundaries seems to be all arcs, is very similar to the section in figure 5. It seems to be that round buildings bring smooth circular spatial boundaries.

Sample 5, consisting of long and thin buildings, brings a special sharp spatial pattern of cells at the gable of each building. It’s mainly because the sharp change of the solid angle at the end of each building. If the thickness of the building continues to decrease, the wall-like artificial entity will bring sharper ends to its corresponding cell.

The last sample consists of two high-rises (with height of 78m and 111m) and four podium buildings of two or three floors. This pair of high-rises absolutely dominate the open space at the ground floor with their huge volume. It leads to a quite different pattern of spatial cells in this sample compared to the others. In the first five samples, all of the spatial cells, which are not adjacent to the boundary \( F \), have more than three neighbors. However, in sample 6, the two-story square building near the center of \( F \) is surrounded by two high-rises. Its spatial cell’s boundary only connected with the cells of the high-rises, which means the space that belongs to it are isolated by the union of neighbourhoods belonging to two high-rises where they always occupy the majority of observer’s field of view compared with the other buildings. It reminds the architects that the strong compressions brought by high-rises may lead to a strong control of open space.

4. CONCLUSIONS

These results we’ve discussed above show us a brand new perspective of spatial cognition. Unlike other spatial studies on the basis of mapping of isovist or the analysis of images taken by camera, the key point of both experiments, whether the entities in them are spheres or 2.5D convex hulls, is to compare the visual impression of different things through their solid angles in the observer’s sight. The reason why we can use solid angle as a measurement of visual perception, is that it truly reflects the nature of how people sense space through their eyes. Bigger solid angle contributes to deeper visual impression and stronger feeling of enclosed. Only with this significant criterion can we achieve the mapping of spatial structures, by introducing the concept of equa-visual set where two or more involved entities appear to be the same size. These cell-like spatial structures reveals the corresponding relationships between certain entity and the subspace which is controlled and created by these entities.

The boundaries (surface \( B \)) separating the open space into cells are the most public districts because all entities adjacent to it get a visual balance here, and all the other entities appearing in sight are relatively weaker in space occupancy. Actually, we can explain the boundaries as a set of shared ‘field lines’ (surfaces in the first model and curves in the second model) of visual perception, referring to the concept of field in physics. We can also use the intensity of visual perception as a measurement of publicity or privacy to describe each spatial cell which belongs to a given entity. There is much more
to the field and intensity of visual perception than we have space to explain here. However, it could probably be a useful reference for designers to set appropriate facilities or constructions in either public or private open space.

The vectorization algorithm ensures the accuracy of the result, especially in calculating the shape of the spatial boundaries, compared with rasterization algorithm. Besides the first experiment with spheres, for example, the step-by-step algorithm in the second model, which is based on human walking pattern, performs better than drawing raster mapping of the solid angles from equidistant sample points arranged in a matrix on the ground plane.

There’re still two more questions to be further discussed. The first one is about the obstructed view of entities. In both experiments, the computation of solid angle from a given point does not exclude the part of an entity that is blocked by the other ones. Even though it’s not easy to prove and evaluate this impact on the final result for now, it’s not hard to see that the shared boundary between two entities in adjacent cells are contributed by themselves rather than other further neighbourhoods due to the majority of solid angle according to equation (1) and (2).

The second problem is on the simplification of the environment in the real world. We only use straight convex hulls to stimulate the buildings in the world we live in on a flat ground. The sky, terrain, plants, animals and other beings are all ignored in the experiment. Even for the buildings, it’s hard to replace all their possible shapes with 2.5D convex hulls. Furthermore, the scaling details, materials, colors, light and shade, which are all important to visual impression, are ignored due to the simplicity.

In the end, what is the use of these virtual experiments and results? Is it only a different perspective of spatial cognition, or just making the perception of the environment more complicated again? Can it really help the designer to improve their spatial planning efficiently and effectively? We are not sure at all. The only thing we know for sure is that we are trying to observe and understand the perceptual process of ourselves which we take for granted in everyday life. We hope this paper will be the starting point for this kind of thinking.

REFERENCES

Benedikt, M., (1979), “To take hold of space: isovist and isovist field”, Environment and Planning B, 6, 47-65
Liu, T., (2011), Vectoring the street spatial outline from streetscape moving images, Master Dissertation, Nanjing: Nanjing University
Peripheral vision, [online] Available at <https://en.wikipedia.org/wiki/Peripheral_vision> (18/08/2018 00:59)
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